TIFR-TH-92-57
hepth@xxx/9210050
October, 1992

BLACK HOLES AND SOLITONS IN STRING THEORY

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ABSTRACT

In this talk, I discuss a general method for constructing classical solutions of the equations of motion arising in the effective low energy string theory, and discuss specific applications of this method.

(Based on talks given at the Johns Hopkins Workshop held at Goteborg, June 8-10, 1992, and ICTP Summer Workshop held at Trieste, July 2-3, 1992)

1. Introduction

In recent years there has been much interest in the construction of classical soliton $[1-4]$ and black hole $[5-9]$ solutions in string theory, including those in low [10] dimensions, and those which correspond to solvable conformal field theories [11]. There are several motivations for studying classical solutions in string theory; I shall only quote two of them here. First of all, one must keep in mind that the full spectrum of string theory should include soliton like solutions that might be present in string theory, and various non-perturbative symmetries in string theory may become manifest only after including all the soliton states in the spectrum. Secondly, since string theory is expected to provide us with a finite, consistent theory of quantum gravity, various puzzles involving black hole evaporation might be resolved by studying them in the context of string theory. This analysis would require construction of black hole solutions in this theory.

In this talk I shall discuss a general method for generating new classical solutions from known solutions $[12-18]$. The general idea is as follows. Let ϕ^i be the fields and $S(\phi^i)$ be the action of a classical field theory, and G be the group of symmetries of the action S. In that case G is also the group of symmetries of the equations of motion, and if $\phi_i^{(cl)}$ is a solution of the equations of motion, then, $\forall g \in G$, $g\phi_i^{(cl)}$ is also a solution of the equations of motion. However, $\phi_i^{(cl)}$ and $g\phi_i^{(cl)}$ are equivalent solutions, in the sense that no physical experiment can distinguish between these two solutions.

In a generic situation, if we restrict ϕ_i to a special class K of field configurations (e.g. field configurations that are independent of d of the dimensions) and write down an expression for the action, the symmetry of the corresponding action will be a subgroup H of G consisting of those elements of G which keep ϕ_i inside the class K. However, in some cases the relevant symmetry group $\mathcal{G}$ may be bigger than H, consisting of elements outside the original symmetry group G. If $\tilde{g}$ denotes such an element of $\mathcal{G}$, and $\phi_i^{(cl)}$ denotes a solution of the equations of motion belonging to the class K, then $\tilde{g}\phi_i^{(cl)}$ denotes a new solution of the equations of motion of the full theory. In this case, however this represents a physically inequivalent solution, since $\tilde{g}$ is not an element of the symmetry group G of the theory.

Thus if string theory happens to possess such extended symmetries when we restrict field configurations to some special classes, we might use them to generate new classical solutions from known ones. I shall show that there are at least two classes of such symmetries in string theory. Both these classes of symmetries were studied in the context of supergravity theories in the past [19] [20]. The first class of symmetries [12]- [16] [21 − 23] allows us, in particular, to construct charged solutions starting from charge neutral ones. The second class of symmetries allows us to construct solutions carrying magnetic charge starting from solutions carrying electric charge [17] [18] [24].

This review will be organised as follows. In sect.2 I shall explain the origin of the first class of symmetries using the language of string field theory and show that they are valid at the string tree level to all orders in α'. In sect.3 I shall see how this symmetry manifests itself in the low energy effective field theory. Sects.4 and 5 will be devoted to specific applications of this transformation towards constructing new classical solutions of string theory. In sect.4 I shall discuss the construction of rotating charged black hole solutions in string theory in four dimensions. Sect.5 will be devoted to the construction of static classical solutions that represent fundamental heterotic string carrying electric charge and electric current. In sect.6 I shall discuss the second class of symmetries − the electric-magnetic duality transformation − of the equations of motion of string theory in four dimensions; and show how, using this transformation, one can construct rotating black hole solutions in string theory carrying both, electric and magnetic charges.

As a guide to the reader, I would suggest that those who are completely unfamiliar with the concept of string field theory may skip section 2; the rest of the article can be read without this section. At the same time, I would like to emphasize that knowledge of string field theory required for reading section 2 is minimal.

2. String Field Theory Argument

I shall first briefly review the formulation of string field theory [25]. For simplicity I shall consider the case of closed bosonic string field theory, although the case of superstring and heterotic string field theory may be dealt with in the same way. Let $\mathcal{H}$ denote the Hilbert space of the combined matter-ghost conformal field theory in two dimensions describing first quantized string theory. The string field can be identified to a state $|\chi\rangle$ in $\mathcal{H}$ of ghost number 2 (with the convention that the SL(2,C) invariant vacuum has ghost number 0), and annihilated by $L_0 - \overline{L}_0$ and $b_0 - \overline{b}_0$. If $\{|\chi_r\rangle\}$ is a basis of such states in $\mathcal{H}$, then we may express $|\chi\rangle$ as $\sum_r \psi_r |\chi_r\rangle$. The ψ_r's are the dynamical variables in string theory (the component fields). With this convention, the kinetic term in string theory is proportional to,

$$\langle\chi|Q_B c_0^-|\chi\rangle = \psi_r \psi_s \langle\chi_r|Q_B c_0^-|\chi_s\rangle \tag{2.1}$$

where Q_B is the BRST charge of the first quantised string theory, and $c_0^- = (c_0 - \overline{c}_0)/\sqrt{2}$. On the other hand, the interaction terms are given by,

$$\sum_{n=3}^{\infty} A_{r_1 \ldots r_n} \psi_{r_1} \ldots \psi_{r_n} \tag{2.2}$$

where $A_{r_1 \ldots r_n}$ are coefficients calculated in terms of correlation functions of certain conformal transform of the field operators $\chi_{r_1}, \ldots \chi_{r_n}$ and ghost fields in the two dimensional conformal field theory. The specific form of $A_{r_1, \ldots r_n}$ will not be required for our analysis.

Let us now consider the situation where the original background around which we are formulating our string field theory is a free field theory of D scalar fields, together with another conformal field theory of central charge $26-D$. Restricting to field configurations independent of d of the D directions would amount to choosing the string field configuration $|\tilde{\chi}\rangle$ which carry zero momentum in these d directions

$X^{D-d+1}, \ldots X^D$. Let $\{|\tilde{\chi}_\alpha\rangle\}$ denote the subset of the basis vectors of $\{|\chi_r\rangle\}$ which carry zero momentum in these d directions. Then $|\tilde{\chi}\rangle$ may be expanded as,

$$|\tilde{\chi}\rangle = \sum_\alpha \tilde{\psi}_\alpha |\tilde{\chi}_\alpha\rangle \tag{2.3}$$

Thus the kinetic term of the restricted action will be given by,

$$\tilde{\psi}_\alpha \tilde{\psi}_\beta \langle\tilde{\chi}_\alpha|Q_B c_0^-|\tilde{\chi}_\beta\rangle \tag{2.4}$$

and the interaction terms are given by,

$$\sum_{n=3}^\infty \tilde{A}_{\alpha_1\ldots\alpha_n} \tilde{\psi}_{\alpha_1} \ldots \tilde{\psi}_{\alpha_n} \tag{2.5}$$

The coefficients $\tilde{A}_{\alpha_1\ldots\alpha_n}$ are now computed in terms of correlation functions of (conformal transforms of) the fields $\tilde{\chi}_{\alpha_1}, \ldots \tilde{\chi}_{\alpha_n}$ and ghost fields in the two dimensional confomal field theory. Since the fields $\tilde{\chi}_{\alpha_i}$ carry zero momentum in the d directions, part of the correlation function involving the fields $X^{D-d+1}, \ldots X^D$ factorises completely into holomorphic and antiholomorphic parts. Each of these parts must be separately invariant under the d dimensional Lorentz transformations.* Thus the net symmetry of the coefficients $\tilde{A}_{\alpha_1\ldots\alpha_n}$ is $O(d)\times O(d)$ (or $O(d-1,1)\times O(d-1,1)$ if one of the d coordinates $X^{D-d+1}, \ldots X^D$ is time-like) [13]. The same result holds for $\langle\tilde{\chi}_\alpha|Q_B c_0^-|\tilde{\chi}_\beta\rangle$. Of these, the diagonal $O(d)$ subgroup is the usual rotational symmetry of the original action. The other generators of the $O(d)\times O(d)$ transformation represent symmetries of the action only for the restricted set of field configurations, and hence may be used to generate new classical solutions of string theory starting from known ones.

$\star$ To see this, let us consider a correlation function of the form $\langle\partial X^\mu(z,\overline{z})\overline{\partial} X^\nu(z,\overline{z})\partial X^\tau(w,\overline{w})\overline{\partial} X^\sigma(w,\overline{w})\rangle$. This correlation function is proportional to $\eta^{\mu\tau}\eta^{\nu\sigma}/|z-w|^4$, and is invariant under separate Lorentz transformations in the indices μ,τ and in the indices ν,σ.

To see explicitly how the $O(d)\times O(d)$ transformation acts on the component fields, let us consider a component field expansion of the form:

$$|\tilde{\chi}\rangle = \int d^{D-d}k\Big[\big(h_{\gamma\delta}(k)+b_{\gamma\delta}(k)\big)\alpha^{\gamma}_{-1}\overline{\alpha}^{\delta}_{-1}c_1\overline{c}_1+\phi(k)(c_1c_{-1}-\overline{c}_1\overline{c}_{-1})\Big]|k^\gamma=0,k^i\rangle+\dots \tag{2.6}$$

where X^γ $(D-d+1\leq\gamma\leq D)$ denote the set of coordinates on which the solution does not depend, X^i $(1\leq i\leq D-d)$ denote the set of coordinates on which the solution does depend, α^γ_{-1}, $\overline{\alpha}^\gamma_{-1}$ are the oscillators appearing in the expansion of the two dimensional fields X^γ, and $\dots$ denote (infinite number of) other terms in the expansion of $|\chi\rangle$ that we have not written down. Since the $O(d)\times O(d)$ transformation corresponds to independent rotation of holomorphic and anti-holomorphic indices, its action on the fields h, b and ϕ is given by,

$$(h+b)\to S(h+b)R^T, \qquad \phi\to\phi \tag{2.7}$$

where S and R are $O(d)$ matrices. The transformations for which $S=R$ correspond to the usual rotation and do not generate inequivalent solutions. Thus all inequivalent solutions may be generated by taking $S=I$, R arbitrary.

The analysis for heterotic and superstring field theories may be carried out in the same way. Since most of our applications will deal with solutions in heterotic string theory, let us discuss this case briefly. In this theory, at least in the sector involving the bosonic fields (Neveu-Schwarz states) the string field theory action may be written in the same form as in eqs.(2.1),(2.2) [26], with the difference that the definition of $A_{r_1\dots r_n}$ also involves some insertion of picture changing operators [27]. Thus the same set of arguments show that if we restrict to field configurations independent of d of the dimensions, the resulting action will have an $O(d)\times O(d)$ symmetry. In this case, however, the symmetry is even larger. To see this, let us note that besides the usual bosonic coordinates, heterotic string theory also has 16 extra bosonic coordinates, which are purely antiholomorphic functions of the two dimensional coordinates. Let us denote these by Y^I, and consider backgrounds where the corresponding state $|\tilde{\chi}\rangle$ carry zero momenta conjugate to p of these 16 coordinates,[†] besides carrying zero momentum conjugate to the d usual bosonic coordinates. Since the correlation function of such states in the two dimensional conformal field theory on the plane is invariant under separate rotation among the d holomorphic and $d+p$ antiholomorphic coordinates, the resulting action will be invariant under $O(d)\times O(d+p)$ transformation. (Although the coordinates Y^I are compact, the correlation functions of the kind we are considering are completely insensitive to this fact, and there is full $O(d+p)$ symmetry involving rotations among the anti-holomorphic components of X^γ and Y^I coordinates.) As before, the diagonal $O(d)$ subgroup generates usual space-time rotation; the rest of the generators can be used to generate new solutions from known ones.

Again to see an example of how this transformation acts on the component fields, let us consider the following component field expansion of the heterotic string field in the Neveu-Schwarz sector:

$$
\begin{aligned}
|\chi\rangle = \int d^{D-d}k\Big[&\big(h_{\gamma\delta}(k)+b_{\gamma\delta}(k)\big)\alpha^\gamma_{-1}\overline{\alpha}^\delta_{-1}c_1\overline{c}_1+\phi(k)(c_1c_{-1}-\overline{c}_1\overline{c}_{-1})\\
&a^I_\gamma(k)\alpha^\gamma_{-1}\overline{\beta}^I_{-1}c_1\overline{c}_1+\ldots\Big]e^{-\phi}(0)|k^\gamma=0,k^i\rangle
\end{aligned}
\tag{2.8}
$$

where $\overline{\beta}^I_n$ denotes the oscillator associated with the internal coordinates Y^I, and ϕ is the bosonized ghost field [27]. The $O(d)\times O(d+p)$ transformation then acts on the oscillators as,

$$
\begin{gathered}
\alpha^\gamma_{-1}\to S_{\gamma'\gamma}\alpha^{\gamma'}_{-1}\\
\begin{pmatrix}\overline{\alpha}^\gamma_{-1}\\ \overline{\beta}^I_{-1}\end{pmatrix}\to R^T\begin{pmatrix}\overline{\alpha}^\gamma_{-1}\\ \overline{\beta}^I_{-1}\end{pmatrix}
\end{gathered}
\tag{2.9}
$$

where S and R are $O(d)$ and $O(d+p)$ matrices respectively. The action of these

† This corresponds to background field configurations which are invariant under a $U(1)^p$ subgroup of the gauge group.

transformations on the fields h, b and a can easily be seen to be of the form:

$$(h+b \quad a) \to S(h+b \quad a)R^T \tag{2.10}$$

where $(h+b \quad a)$ is regarded as a $d \times (d+p)$ matrix.

Sometimes we may start with a solution that is invariant under one or more space-time supersymmetry transformations. Thus it is natural to ask if the transformed solution is also invariant under such a symmetry. A general string field theoretic argument showing that this must be so may be given as follows. Since so far we do not have a consistent closed heterotic string field theory (although we do have such a theory involving only the Neveu-Schwarz states [26]) we do not know precisely how the space-time supersymmetry operator will look like in the string field theory. However, from the general analysis [27] it is clear that space-time supersymmetry will act only on the holomorphic part of the vertex operators representing a general off-shell string field configuration. Since according to the arguments given above the $O(d+p-1,1)$ symmetry transformation induced by the matrix R acts on the anti-holomorphic part of the vertex operators, these two symmetry transformations commute. On the other hand, the $O(d-1,1)$ transformation induced by the matrix S can be regarded as a combination of usual Lorentz transformation, and an $O(d+p-1,1)$ transformation. Thus the effect of an S transformation will be a usual Lorentz transformation of the supersymmetry transformation parameter. As a result, the $O(d-1,1) \times O(d+p-1,1)$ transformation will transform a supersymmetric solution to a supersymmetric solution, with the supersymmetry transformation parameter being Lorentz rotated by the $O(d-1,1)$ component.

Before I conclude this section, I wish to emphasise that these results are exact for the tree level string field theory, and no α' expansion has been made.

3. Effective Field Theory Analysis

From the analysis of the last section it follows that the low energy effective field theory describing heterotic string theory should also possess the $O(d)\times O(d+p)$ symmetry if we restrict our field configuration to backgrounds which are independent of d of the D coordinates and is invariant under a $U(1)^p$ subgroup of the gauge group. In fact if we knew the exact relationship between the component fields that appear in the expansion of the string field, and those that appear in the low energy effective field theory, we could find out how the $O(d)\times O(d+p)$ transformation acts on the fields appearing in the effective field theory, since we already know how it acts on the component fields of string field theory. Unfortunately, the exact relationship between the two sets of fields is not known, the only known result is in the weak field limit. Using this, we may express the metric $G_{\mu\nu}$, the dilaton Φ, the antisymmetric tensor field $B_{\mu\nu}$, and the gauge field A^I_μ appearing in the effective field theory in terms of string field components as,

$$\begin{aligned} G_{\mu\nu} =& \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(\chi^2) \\ B_{\mu\nu} =& b_{\mu\nu} + \mathcal{O}(\chi^2) \\ \Phi =& \phi + \frac{1}{2} h_{\mu\nu}\eta^{\mu\nu} + \mathcal{O}(\chi^2) \\ A^I_\mu =& a^I_\mu + \mathcal{O}(\chi^2) \end{aligned} \tag{3.1}$$

Thus the transformation laws of the fields $G_{\mu\nu}$, $B_{\mu\nu}$, A^I_μ and Φ are known explicitly in the weak field limit. The full $O(d+p)\times O(d)$ transformation laws of these fields are found by explicitly examining the low energy effective action of the heterotic string theory. This action is given by,

$$\begin{aligned} S = -\int d^D x \sqrt{-\det G}\, e^{-\Phi} \Big[& - R^{(D)}(G) + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \\ & - G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}\sum_{I=1}^{p} F^I_{\mu\nu} F^{I\mu\nu} + \ldots \Big] \end{aligned} \tag{3.2}$$

where $\ldots$ denote terms in the effective action involving higher derivative terms, and other fields which are set to zero for the particular class of backgrounds we are considering, $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$, $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho}$ + cyclic permutations $-(\Omega^{(3)}(A))_{\mu\nu\rho}$, $R^{(D)}$ denotes the D dimensional Ricci scalar, and $\Omega_3(A)$ is the Chern-Simons 3-form, defined by,

$$\Omega_3(A) = \frac{1}{4}(A^I_\mu F^I_{\nu\rho} + \textit{ cyclic permutations}) \tag{3.3}$$

In writing down the action (3.2) we have restricted ourselves to field configurations where the background gauge fields are fully abelian, p denotes the number of unbroken U(1) gauge groups after compactification. We shall further restrict to field configurations where the background gauge fields are independent of d of the D coordinates. For such restricted set of field configurations, let us define the matrix:

$$\mathcal{K}_{\mu\nu} = -G_{\mu\nu} - B_{\mu\nu} - \frac{1}{4}A^I_\mu A^I_\nu \tag{3.4}$$

and treat $G_{\mu\nu}$ and A^I_μ as $D \times D$ and $D \times p$ matrices respectively. In terms of these matrices we define a $(2D + p) \times (2D + p)$ matrix as,

$$\mathcal{M} = \begin{pmatrix} (\mathcal{K}^T - \eta)G^{-1}(\mathcal{K} - \eta) & (\mathcal{K}^T - \eta)G^{-1}(\mathcal{K} + \eta) & -(\mathcal{K}^T - \eta)G^{-1}A \\ (\mathcal{K}^T + \eta)G^{-1}(\mathcal{K} - \eta) & (\mathcal{K}^T + \eta)G^{-1}(\mathcal{K} + \eta) & -(\mathcal{K}^T + \eta)G^{-1}A \\ -A^T G^{-1}(\mathcal{K} - \eta) & -A^T G^{-1}(\mathcal{K} + \eta) & A^T G^{-1} A \end{pmatrix} \tag{3.5}$$

where ^{T} denotes transposition of the matrix, and η is the D dimensional Minkowski metric. It is easy to see that for a given $\mathcal{M}$, the fields $G_{\mu\nu}$, $B_{\mu\nu}$ and A^I_μ are completely determined. If we assume that the background field configuration is restricted to be independent of the last d of the D coordinates $X^{D-d+1}, \ldots X^D$, then the action for this restricted field configuration can be shown to be invariant under the following transformation of the fields [14]:

$$\mathcal{M} \to \mathcal{M}' = \Omega \mathcal{M} \Omega^T, \quad \Phi \to \Phi' = \Phi + \frac{1}{2}(\ln \det G' - \ln \det G) \tag{3.6}$$

where,

$$\Omega = \begin{pmatrix} I_{D-d} & & & \\ & S & & \\ & & I_{D-d} & \\ & & & R \end{pmatrix} \tag{3.7}$$

S and R are $O(d)$ and $O(d+p)$ matrices respectively, and I_n is the n dimensional identity matrix. (If the d coordinates on which the solution does not depend includes the time coordinate, then S and R are $O(d-1,1)$ and $O(d+p-1,1)$ matrices respectively.) Note that $\mathcal{M}$ contains full information about the fields $G_{\mu\nu}$, $B_{\mu\nu}$ and A^I_μ; hence specifying the transformation laws of $\mathcal{M}$ and Φ automatically specifies the transformation laws of all the fields in the theory. In order to see that this transformation is the same as the $O(d) \times O(d+p)$ transformation discussed in the previous section, we can verify that they agree with the transformation laws derived in the last section in the weak field limit.

Using the transformation laws given in eqs.(3.6), we can generate new solutions of the equations of motion from the known solutions that are independent of some of the coordinates. In the next two sections we shall see some specific applications of this procedure of generating new solutions of the equations of motion.

4. Rotating Charged Black Holes in Four Dimensions

In order to apply the method outlined in the last section for generating new solutions of the equations of motion, we need to have at least one known solution of these equations. As we shall now see, such solutions are provided by already known solutions of Einstein's equation in the absence of any matter field. To see this let us write down the equations of motion following from the action given in eq.(3.2). They are,

$$
\begin{aligned}
&R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + D_\mu D_\nu \Phi - G_{\mu\nu}D^\rho D_\rho \Phi + \frac{1}{2}G_{\mu\nu}D_\rho \Phi D^\rho \Phi \\
&- \frac{1}{4}(H_{\mu\rho\tau}H_\nu{}^{\rho\tau} - \frac{1}{16}H_{\rho\sigma\tau}H^{\rho\sigma\tau}G_{\mu\nu}) - \frac{1}{4}(F^I_{\mu\rho}F^I_\nu{}^\rho - \frac{1}{4}G_{\mu\nu}F^I_{\rho\tau}F^{I\rho\tau}) \\
=&0
\end{aligned}
\tag{4.1}
$$

$$
D_\mu(e^{-\Phi}H^{\mu\nu\rho}) = 0 \tag{4.2}
$$

$$
D_\mu(e^{-\Phi}F^{I\mu\nu}) + \frac{1}{2}e^{-\Phi}H_{\rho\mu}{}^\nu F^{I\rho\mu} = 0 \tag{4.3}
$$

$$
R - D_\mu \Phi D^\mu \Phi - \frac{1}{12}H_{\rho\sigma\tau}H^{\rho\sigma\tau} - \frac{1}{8}F^I_{\rho\tau}F^{I\rho\tau} + 2D^\mu D_\mu \Phi = 0 \tag{4.4}
$$

From eqs.(4.1)-(4.4) we see that if we set

$$
A^I_\mu = B_{\mu\nu} = \Phi = 0 \tag{4.5}
$$

then all the equations are satisfied if the metric satisfies the Einstein's equation $R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 0$. In other words, given a solution of the pure Einstein's equation, we can construct a solution of the full string theory equations of motion by setting $A^I_\mu = B_{\mu\nu} = \Phi = 0$.

We now restrict ourselves to four dimensions, and consider the most general static black hole solution of Einstein's equation. This is given by the Kerr solution:

$$
\begin{aligned}
ds^2 =& -\frac{\rho^2 + a^2\cos^2\theta - 2m\rho}{\rho^2 + a^2\cos^2\theta}dt^2 + \frac{\rho^2 + a^2\cos^2\theta}{\rho^2 + a^2 - 2m\rho}d\rho^2 + (\rho^2 + a^2\cos^2\theta)d\theta^2 \\
&+ \frac{\sin^2\theta}{\rho^2 + a^2\cos^2\theta}\{(\rho^2 + a^2)(\rho^2 + a^2\cos^2\theta) + 2m\rho a^2\sin^2\theta\}d\phi^2 \\
&- \frac{4m\rho a\sin^2\theta}{\rho^2 + a^2\cos^2\theta}dtd\phi
\end{aligned}
\tag{4.6}
$$

This metric, together with the field configuration given in eq.(4.5), gives a solution of the equations of motion (4.1)-(4.4).

We now note that the solution is independent of the time coordinate. Hence we can generate new solutions by making a transformation of the form given in eqs.(3.6), (3.7), with,

$$S = I_p, \qquad R = \begin{pmatrix} \cosh\alpha & \sinh\alpha & \\ \sinh\alpha & \cosh\alpha & \\ & & I_{p-1} \end{pmatrix} \tag{4.7}$$

After some algebraic manipulations, the transformed solution may be expressed as [28],

$$\begin{aligned} ds'^2 = &- \frac{(\rho^2 + a^2\cos^2\theta - 2m\rho)(\rho^2 + a^2\cos^2\theta)}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} dt^2 \\ &+ \frac{\rho^2 + a^2\cos^2\theta}{\rho^2 + a^2 - 2m\rho} d\rho^2 + (\rho^2 + a^2\cos^2\theta) d\theta^2 \\ &+ \{(\rho^2 + a^2)(\rho^2 + a^2\cos^2\theta) + 2m\rho a^2\sin^2\theta + 4m\rho(\rho^2 + a^2)\sinh^2\frac{\alpha}{2} \\ &+ 4m^2\rho^2\sinh^4\frac{\alpha}{2}\} \times \frac{(\rho^2 + a^2\cos^2\theta)\sin^2\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} d\phi^2 \\ &- \frac{4m\rho a\cosh^2\frac{\alpha}{2}(\rho^2 + a^2\cos^2\theta)\sin^2\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} dt d\phi \end{aligned} \tag{4.8}$$

$$\Phi' = -\ln\frac{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}}{\rho^2 + a^2\cos^2\theta} \tag{4.9}$$

$$A'_\phi = -\frac{2m\rho a\sinh\alpha\sin^2\theta}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} \tag{4.10}$$

$$A'_t = \frac{2m\rho\sinh\alpha}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} \tag{4.11}$$

$$B'_{t\phi} = \frac{2m\rho a\sinh^2\frac{\alpha}{2}\sin^2\theta}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} \tag{4.12}$$

The other components of A'_μ and $B'_{\mu\nu}$ vanish. (Here A_μ denotes the component

A^1_μ.) The Einstein metric (defined as $ds'^2_E \equiv e^{-\Phi'} ds'^2$) is given by,

$$
\begin{aligned}
ds'^2_E = & -\frac{\rho^2 + a^2\cos^2\theta - 2m\rho}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} dt^2 + \frac{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}}{\rho^2 + a^2 - 2m\rho} d\rho^2 \\
& + (\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}) d\theta^2 - \frac{4m\rho a\cosh^2\frac{\alpha}{2}\sin^2\theta}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} dt d\phi \\
& + \{(\rho^2 + a^2)(\rho^2 + a^2\cos^2\theta) + 2m\rho a^2\sin^2\theta + 4m\rho(\rho^2 + a^2)\sinh^2\frac{\alpha}{2} \\
& + 4m^2\rho^2\sinh^4\frac{\alpha}{2}\} \times \frac{\sin^2\theta}{\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2}} d\phi^2
\end{aligned}
\tag{4.13}
$$

The various field strengths associated with the gauge fields are given by,

$$
F_{\rho\phi} = \frac{2ma\sinh\alpha\sin^2\theta(\rho^2 - a^2\cos^2\theta)}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{4.14}
$$

$$
F_{\theta\phi} = -\frac{4m\rho a\sinh\alpha(\rho^2 + a^2 + 2m\rho\sinh^2\frac{\alpha}{2})\sin\theta\cos\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{4.15}
$$

$$
F_{\rho t} = -\frac{2m\sinh\alpha(\rho^2 - a^2\cos^2\theta)}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{4.16}
$$

$$
F_{\theta t} = \frac{4m\rho a^2\sinh\alpha\sin\theta\cos\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{4.17}
$$

$$
e^{-\Phi}\sqrt{-G}H^{\rho t\phi} = \frac{2ma\sinh^2\frac{\alpha}{2}(\rho^2 - a^2\cos^2\theta)\sin\theta}{(\rho^2 + a^2\cos^2\theta)^2} \tag{4.18}
$$

$$
e^{-\Phi}\sqrt{-G}H^{\theta\phi t} = \frac{4m\rho a\sinh^2\frac{\alpha}{2}\cos\theta}{(\rho^2 + a^2\cos^2\theta)^2} \tag{4.19}
$$

By examining the asymptotic properties of the solution we can easily determine that this solution describes a rotating object with mass M, angular momentum J, charge Q and magnetic moment μ, given by,[⋆]

$$
\begin{aligned}
M =& \frac{m}{2}(1+\cosh\alpha), \qquad Q = \frac{m}{\sqrt{2}}\sinh\alpha \\
J =& \frac{ma}{2}(1+\cosh\alpha), \qquad \mu = \frac{1}{\sqrt{2}} ma\sinh\alpha
\end{aligned}
\tag{4.20}
$$

Hence the g factor is given by,

$$g \equiv \frac{2\mu M}{QJ} = 2 \tag{4.21}$$

For $a = 0$, the above solution reduces to the charged black hole solution of refs.[6][7]. Construction of rotating charged black hole solution in string theory in the limit of small angular momentum had been discussed previously by Horne and Horowitz [29].

By examining the above solution, we see that it has two horizons at,

$$\rho = m \pm \sqrt{m^2 - a^2} = M - \frac{Q^2}{2M} \pm \sqrt{(M - \frac{Q^2}{2M})^2 - \frac{J^2}{M^2}} \tag{4.22}$$

These event horizons exist as long as,

$$M - \frac{Q^2}{2M} \geq \frac{|J|}{M} \tag{4.23}$$

Thus the extremal limit, the limit in which the event horizon is about to disappear, is given by,

$$M^2 \to \frac{Q^2}{2} + |J| \tag{4.24}$$

The area of the outer event horizon, which is proportional to the entropy of the black hole, is given by,

$$A = 8\pi M\left(M - \frac{Q^2}{2M} + \sqrt{(M - \frac{Q^2}{2M})^2 - \frac{J^2}{M^2}}\right) \tag{4.25}$$

Thus in the extremal limit it approaches the value $8\pi|J|$.

⋆ A factor of $2\sqrt{2}$ in the definition of the electric charge and electric current has been introduced so that our normalization matches that of ref.[7].

Finally, the surface gravity of the black hole, which is proportional to the Hawking temperature of the black hole, is given by,

$$\frac{\sqrt{(2M^2-Q^2)^2-4J^2}}{2M(2M^2-Q^2+\sqrt{(2M^2-Q^2)^2-4J^2})} \tag{4.26}$$

Note that the surface gravity vanishes in the extremal limit.

5. Macroscopic Charged Heterotic String

We now turn to the second example. In this case we start from a solution of the string theory equations of motion given in ref.[1], describing fundamental heterotic string in D dimensions. The solution is given by,

$$\begin{aligned} ds^2 =& e^E\{-dt^2+(dx^{D-1})^2\}+\sum_{i=1}^{D-2}dx^i dx^i \\ B_{(D-1)t} =& 1-e^E, \quad \Phi = E, \quad A^I_\mu = 0 \end{aligned} \tag{5.1}$$

where,

$$e^{-E} = 1 + MG^{(D-2)}(\vec{r}) \tag{5.2}$$

$\vec{r}$ denotes the $D-2$ dimensional vector $(x^1,\ldots x^{D-2})$, and $G^{(D-2)}$ is the $D-2$ dimensional Green's function, given by,

$$\begin{aligned} G^{(D-2)}(\vec{r}) =& \frac{1}{(D-4)\omega_{D-3}r^{D-4}} \quad \text{for } D>4 \\ =& -\frac{1}{2\pi}\ln r \quad \text{for } D=4 \end{aligned} \tag{5.3}$$

where ω_{D-3} is the volume of a unit $D-3$ sphere. M is an arbitrary constant. By looking at the source terms at $\vec{r}=0$ that this solution corresponds to, we can identify the solution to be the field around a fundamental heterotic string [1] sitting at $\vec{r}=0$ in the gauge $X^0=\tau$, $X^{D-1}=\sigma$, $(\sqrt{-\gamma})^{-1}\gamma_{mn}=\eta_{mn}$, provided we identify the constant M appearing in the solution to the string tension. (Here γ_{mn} denotes the world sheet metric).

By examining the solution we see that it is independent of the coordinates t and x^{D-1}. Hence we can generate new solutions by performing transformations of the kind discussed in sec.3. We choose a transformation matrix Ω of the form:

$$\Omega = \begin{pmatrix} I_{2D-1} & & & \\ & \cosh\alpha & \sinh\alpha & \\ & \sinh\alpha & \cosh\alpha & \\ & & & I_{p-1} \end{pmatrix} \tag{5.4}$$

Using eq.(3.6), and some algebra, we find the transformed solution to be [30]:

$$\begin{aligned} ds^2 =& \frac{1}{1+NG^{(D-2)}(\vec{r})}(-dt^2 + (dx^{D-1})^2) + \frac{q^2 G^{(D-2)}(\vec{r})}{4N(1+NG^{(D-2)}(\vec{r}))^2}(dt + dx^{D-1})^2 \\ &+ \sum_{i=1}^{D-2} dx^i dx^i \\ B_{(D-1)t} =& \frac{NG^{(D-2)}(\vec{r})}{1+NG^{(D-2)}(\vec{r})} \\ A^1_{D-1} =& A^1_t = \frac{qG^{(D-2)}(\vec{r})}{1+NG^{(D-2)}(\vec{r})} \\ \Phi =& -\ln(1+NG^{(D-2)}(\vec{r})) \end{aligned} \tag{5.5}$$

where

$$N = M\cosh^2\frac{\alpha}{2}, \qquad q = M\sinh\alpha \tag{5.6}$$

The various field strength tensors may be calculated from this solution and we get the following results,

$$\begin{aligned} F^1_{r(D-1)} =& F^1_{rt} = -\frac{q}{r^{D-3}\omega_{D-3}(1+NG^{(D-2)}(\vec{r}))^2} \\ H_{r(D-1)t} =& -\frac{N}{r^{D-3}\omega_{D-3}(1+NG^{(D-2)}(\vec{r}))^2} \end{aligned} \tag{5.7}$$

Note that $\Omega_3(A)$ vanishes everywhere for the specific solution we have constructed. For $D > 4$ the metric is asymptotically flat, and the electric charge per unit length Q, the electric current J and the axionic charge Z associated with the solution may be defined in terms of the asymptotic behavior of the field strengths in the $r \to \infty$ limit as follows:

$$\begin{aligned}
\frac{1}{2\sqrt{2}} F^1_{rt} &\simeq -\frac{Q}{r^{D-3}\omega_{D-3}} \\
\frac{1}{2\sqrt{2}} F^1_{r(D-1)} &\simeq -\frac{J}{r^{D-3}\omega_{D-3}} \\
H_{r(D-1)t} &\simeq -\frac{Z}{r^{D-3}\omega_{D-3}}
\end{aligned} \tag{5.8}$$

Eqs.(5.7) and (5.8) give,

$$Q = \frac{q}{2\sqrt{2}}, \qquad J = \frac{q}{2\sqrt{2}}, \qquad Z = N \tag{5.9}$$

Note that the gauge field configurations associated with the solution describe a radial electric field and an azimuthal magnetic field, which are equal in magnitude. This, in turn, shows that the solution corresponds to a charge and current carrying string, for which the electric current is equal to the charge per unit length of the string.

Analysis of the source terms associated with the solution near the origin $\vec{r} = 0$ reveals that the new solution also describes a fundamental string with string tension N, carrying a world sheet current j^m satisfying $j^0 = -j^1$. This is in accordance with the well known property of the heterotic string that the world-sheet currents in this theory responsible for the gauge symmetry are chiral in nature.

Before concluding this section let us mention some special properties of the solution. The original solution of ref.[1] was shown to be invariant under half of the space-time supersymmetry transformations of the theory. The general argument given in section 2 then shows that the transformed solution should also be invariant under half of the supersymmetry transformations. In fact, since in the present case, the transformation is generated solely by the $O(d+p-1,1)$ part (we see from eqs.(3.7) and (5.4) that $S=I$ in the present case), we expect from the general arguments that the transformed solution will be invariant under the same supersymmetry transformations as the original solution. These results can be verified by explicit calculation. In fact, recently the charged string solution has been reconstructed by demanding that the final solution is supersymmetry invariant [31].

It was shown that the solution of ref.[1] can be regarded as the extremal limit of a black string solution [7]. The present solution can also be shown to be the extremal limit of a charged black string solution [30]. One can simply start from the black string solution of ref.[7] and transform it by the $O(d-1,1)\times O(d+p-1,1)$ transformation to find out the black string solution whose extremal limit is the solution given above.

The authors of ref.[1] also constructed static multi-string solution, representing parallel strings at rest. An $O(d-1,1)\times O(d+p-1,1)$ transformation their solution gives a static, charged, multi-string solution. This, however, is not the most general charged multi-string solution, since the charge densities carried by all the strings are parametrized by a single parameter (the boost angle α appearing in eq.(5.4)). However, a slight modification of the solution thus obtained allows us to construct charged multi-string solutions, where the charge densities carried by different strings are independent of each other, and can also point in arbitrary directions in the p dimensional space corresponding to the p $U(1)$ gauge groups of the theory. This general solution is given by [30],

$$
\begin{aligned}
ds^2 =& \frac{1}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}(-dt^2+(dx^{D-1})^2)\\
&+g(\vec{r})(dt+dx^{D-1})^2+\sum_{i=1}^{D-2} dx^i dx^i\\
B_{(D-1)t} =& \frac{N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}\\
A^I_{D-1} =& A^I_t = \frac{\sum_l q_l^{(I)} G^{(D-2)}(\vec{r}-\vec{r}_l)}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}\\
\Phi =& -\ln(1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l))
\end{aligned}
\tag{5.10}
$$

where,

$$
g(\vec{r}) = \frac{1}{4}\Bigg[\frac{\sum_{l,I}(q_l^{(I)})^2 G^{(D-2)}(\vec{r}-\vec{r}_l)}{N\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)} - \frac{\sum_I\big(\sum_l q_l^{(I)} G^{(D-2)}(\vec{r}-\vec{r}_l)\big)^2}{\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)^2}\Bigg] \tag{5.11}
$$

6. Black Holes Carrying Electric and Magnetic Charges

Besides the $O(d)\times O(d+p)$ invariance for restricted class of backgrounds of the form discussed above, in four dimensions the equations of motion derived from the action (3.2) have another symmetry, known as the electric-magnetic duality symmetry [19],[20]. Using this symmetry one can generate solutions carrying both electric and magnetic charges starting from solutions carrying only electric charge [17] [18] [24]. Unlike the $O(d)\times O(d+p)$ symmetry discussed in the previous sections, the transformation laws under this symmetry take simple form in terms of the Einstein metric $G_{E\mu\nu} = e^{-\Phi}G_{\mu\nu}$. For convenience of writing, throughout this section we shall use the notation $G_{\mu\nu}$ for Einstein metric, and all the indices will be raised and lowered with this metric rather than the σ-model metric used in the last sections.

In order to understand the action of this symmetry, we first note that for any solution of the equations of motion, eq.(4.2) allows us to define a field Ψ as follows:

$$H^{\mu\nu\rho} = -(\sqrt{-\det G})^{-1} e^{2\Phi} \epsilon^{\mu\nu\rho\sigma} \partial_\sigma \Psi \tag{6.1}$$

(Note that the $G_{\mu\nu}$ ($H^{\mu\nu\rho}$) appearing in the above equation is related to $G_{\mu\nu}$ ($H^{\mu\nu\rho}$) appearing in eq.(4.2) by a multiplicative factor of $e^{-\Phi}$ ($e^{3\Phi}$); this is the reason for the appearance of the factor of $e^{2\Phi}$ on the right hand side of eq.(6.1) instead of e^{Φ}.) Let us define,

$$\tilde{F}^{I\mu\nu} = \frac{1}{2}\sqrt{-\det G}\epsilon^{\mu\nu\rho\sigma} F^I_{\rho\sigma} \tag{6.2}$$

The bianchi identity of the field $H_{\mu\nu\rho}$,

$$(\sqrt{-\det G})^{-1} \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} = -\frac{3}{4} F^I_{\mu\nu} \tilde{F}^{I\mu\nu} \tag{6.3}$$

may then be written as,

$$D^\mu (e^{2\Phi} D_\mu \Psi) = \frac{1}{8} \sum_{I=1}^{p} F^I_{\mu\nu} \tilde{F}^{I\mu\nu} \tag{6.4}$$

Let us now define a complex field λ as,

$$\lambda = \Psi + i e^{-\Phi} \equiv \lambda_1 + i\lambda_2 \tag{6.5}$$

The equations of motion (4.1), (4.3), (4.4), and (6.4) may then be written as,

$$R_{\mu\nu} = \frac{\partial_\mu \overline{\lambda} \partial_\nu \lambda + \partial_\nu \overline{\lambda} \partial_\mu \lambda}{4(\lambda_2)^2} + \frac{1}{4}\lambda_2 F^I_{\mu\rho} F^{I\ \rho}_{\nu} - \frac{1}{16}\lambda_2 G_{\mu\nu} F^I_{\rho\sigma} F^{I\rho\sigma} \tag{6.6}$$

$$\frac{D^\mu D_\mu \lambda}{(\lambda_2)^2} + i \frac{D_\mu \lambda D^\mu \lambda}{(\lambda_2)^3} - \frac{i}{16} F^I_{-\mu\nu} F^{I\ \mu\nu}_{-} = 0 \tag{6.7}$$

$$D_\mu(\lambda F_+^{I\ \mu\nu} - \overline{\lambda} F_-^{I\ \mu\nu}) = 0 \tag{6.8}$$

where,

$$F_\pm^I = F^I \pm i\tilde{F}^I \tag{6.9}$$

In terms of the fields $F_\pm^I$, the Bianchi identity for $F_{\mu\nu}^I$ takes the form:

$$D_\mu(F_+^{I\ \mu\nu} - F_-^{I\ \mu\nu}) = 0 \tag{6.10}$$

We now note that eqs.(6.6)-(6.8), and (6.10) are invariant under the following transformations:

$$\lambda \to \lambda + c \tag{6.11}$$

where c is a real number, and,

$$\lambda \to -\frac{1}{\lambda}, \qquad F_+^I \to -\lambda F_+^I, \qquad F_-^I \to -\overline{\lambda} F_-^I \tag{6.12}$$

Invariance of all the equations under (6.11) is manifest. Under (6.12), eq.(6.7) is invariant, eqs.(6.8) and (6.10) get interchanged, and eq.(6.6) transforms to itself plus an extra term, given by,

$$-\frac{\lambda_1(\lambda_2)^2}{|\lambda|^2}(2F_{\mu\rho}^I \tilde{F}_\nu^{I\ \rho} + 2F_{\nu\rho}^I \tilde{F}_\mu^{I\ \rho} - g_{\mu\nu} F_{\rho\tau}^I \tilde{F}^{I\rho\tau}) \tag{6.13}$$

The term given in eq.(6.13), however, vanishes identically in four dimensions, showing that (6.12) is a genuine symmetry of the equations of motion. The two transformations together generate the full SL(2,$\mathcal{R}$) group under which $\lambda \to (a\lambda+b)/(c\lambda+d)$ with $ad - bc = 1$, and $F_+^I \to -(c\lambda + d)F_+^I$.

Applying this 'symmetry' transformation on the known solutions of the equations of motion, we can find new solutions. Let us now apply this transformation to the rotating charged black hole solution given in sect.4. From eqs.(4.18), (4.19), and (6.1) we get,

$$\Psi = \frac{2ma\sinh^2\frac{\alpha}{2}\cos\theta}{\rho^2 + a^2\cos^2\theta} \tag{6.14}$$

(Note that we must multiply G by appropriate factor of e^{Φ} before comparing eqs.(4.18), (4.19) with (6.1).) This gives,

$$\lambda = \frac{2ma\sinh^2\frac{\alpha}{2}\cos\theta + i(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})}{\rho^2 + a^2\cos^2\theta} \tag{6.15}$$

The dual of various field strengths $F_{\mu\nu}$ are given by,

$$\tilde{F}_{\rho\phi} = \frac{4m\rho a^2\sinh\alpha\sin^2\theta\cos\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{6.16}$$

$$\tilde{F}_{\theta t} = -\frac{2ma\sinh\alpha(\rho^2 - a^2\cos^2\theta)\sin\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{6.17}$$

$$\tilde{F}_{\theta\phi} = \frac{2m\sinh\alpha\sin\theta(\rho^2 - a^2\cos^2\theta)(\rho^2 + a^2 + 2m\rho\sinh^2\frac{\alpha}{2})}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{6.18}$$

$$\tilde{F}_{\rho t} = -\frac{4m\rho a\sinh\alpha\cos\theta}{(\rho^2 + a^2\cos^2\theta + 2m\rho\sinh^2\frac{\alpha}{2})^2} \tag{6.19}$$

We can now generate a new solution by performing an SL(2,$\mathcal{R}$) transformation on the above solution. We consider the SL(2,$\mathcal{R}$) transformation $\lambda \to -(1+c^2)/(\lambda+c)$, $F_+ \to -(\lambda + c)F_+/\sqrt{1+c^2}$. The transformed solution is given by,

$$\lambda' = -\frac{1+c^2}{\lambda + c}, \qquad ds'^2 = ds'^2_E, \qquad F'_{\mu\nu} = -\frac{\Psi + c}{\sqrt{1+c^2}}F_{\mu\nu} + \frac{e^{-\Phi}}{\sqrt{1+c^2}}\tilde{F}_{\mu\nu} \tag{6.20}$$

where λ, ds'^2_E, $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are given in eqs.(6.15), (4.13), (4.14)-(4.17), and (6.16)-(6.19) respectively. I shall not write out the solution in detail. The leading components of the electromagnetic field at large ρ are given by,

$$F'_{\rho t} \simeq \frac{2mc\sinh\alpha}{\sqrt{1+c^2\rho^2}}, \qquad F'_{\theta\phi} = \frac{2m\sinh\alpha\sin\theta}{\sqrt{1+c^2}} \tag{6.21}$$

With appropriate normalization (which sets the coefficient of F^2 term in the action to unity), the electric and magnetic charges carried by the solution may be identified to,

$$Q_{el} = \frac{1}{\sqrt{2}}\frac{mc\sinh\alpha}{\sqrt{1+c^2}}, \qquad Q_{mag} = \frac{1}{\sqrt{2}}\frac{m\sinh\alpha}{\sqrt{1+c^2}} \tag{6.22}$$

Since the metric remains the same, the expressions for the mass M and angular momentum J of the black hole in terms of the parameters m, a and α remain the same as in eq.(4.20). Also, all geometric properties of the solution remain the same as in the case of rotating charged black hole. The only difference that occurs in eqs.(4.22)-(4.26) is the replacement of Q^2 by $(Q_{el})^2 + (Q_{mag})^2$. Thus this solution represents a rotating black hole solution carrying both electric and magnetic charge [18]. To linear order in the electric and magnetic charges, this solution was previously constructed in ref.[32].

7. Summary

In this review I have discussed the construction of various black hole and soliton type solutions in string theory using a general method, which relies on the existence of extended symmetries of the equations of motion when we restrict the field configuration to a certain class. Two such extended symmetries have been discussed, one of which appears when we restrict field configurations to be independent of certain directions, and no charged fields are present; the other appears when we restrict the field configurations to four dimensions, and again no charged fields are present. Combining these two symmetry transformations we can generate solutions carrying both electric and magnetic charges by starting from a solution of vacuum Einstein's equation. The method is quite powerful, and using this method one can also generate most of the known black hole type solutions in critical string theory (where the dilaton approaches a constant value asymptotically) by starting from a known solution of vacuum Einstein's equation in some dimension and then repeatedly applying these symmetry transformations.

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